

試求通過  $O(0,0,0)$ ,  $A(4,-2,2)$ ,  $B(2,-4,4)$ ,  $C(3,0,1)$  四點的球面方程式。

詳解：設所求之球面方程式為  $S: x^2 + y^2 + z^2 + dx + ey + fz + g = 0$

球面  $S$  過  $O, A, B, C$  四點，將此四點坐標分別代入球面方程式  $S$

$$\text{可得} \begin{cases} g = 0 \\ 4d - 2e + 2f + g + 24 = 0 \\ 2d - 4e + 4f + g + 36 = 0 \\ 3d + f + g + 10 = 0 \end{cases} \Rightarrow \begin{cases} 4d - 2e + 2f + 24 = 0 \\ 2d - 4e + 4f + 36 = 0 \\ 3d + f + 10 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} 4d - 2e + 2f + 24 = 0 \cdots \cdots (1) \\ d - 2e + 2f + 18 = 0 \cdots \cdots (2) \\ 3d + f + 10 = 0 \cdots \cdots (3) \end{cases}$$

$$(1) - (2) \text{ 可得 } 3d = -6, d = -2$$

$$\text{代入}(3) \text{ 得 } 3 \cdot (-2) + f + 10 = 0, f = -4$$

$$\text{代入}(1) \text{ 得 } 2 \cdot (-2) - e + (-4) + 12 = 0, e = 12 - 4 - 4 = 4$$

$$\Rightarrow g = 0, d = -2, e = 4, f = -4$$

$$\text{故所求球面方程式 } S: x^2 + y^2 + z^2 - 2x + 4y - 4z = 0$$

